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Modeling and Energy Consumption of Unilateral Heating Process of Flat Wood Details

Modeliranje i potrošnja energije u procesu jednostranog zagrijavanja ravnih drvenih elemenata

ABSTRACT • *A methodology has been suggested for mathematical modeling and research of two mutually connected problems: the temperature distribution along the thickness of flat wood details subjected to unilateral heating and the energy consumption of this process. For the realization of the methodology, a 1-dimensional mathematical model has been created and solved for the transient linear heat conduction in flat wood details during their unilateral heating at arbitrary initial and boundary conditions encountered in the practice. Based on the integration of the model's solutions, a numerical approach has been suggested for the computation of the specific energy consumption (for 1 m^2) and the specific heat flux needed for the heating of the details and for the covering of their heat emission in the surrounding environment during unilateral heating aimed at wood plasticizing and bending. This paper presents solutions of the model concerning the non-stationary temperature distribution along the thickness of spruce details with thicknesses of 6, 8 and 10 mm and the non-stationary change in the specific energy consumption and in the specific heat flux during unilateral heating at temperatures of the electrically heated metal band equal to 100, 120, and 140 ℃ aimed at plasticizing and bending of the details in the production of outside curved parts of the body of string musical instruments. The obtained results can be used for technological and energy calculations and for analysis of processes of unilateral heating of wood details at different boundary conditions, as well as in software for systems for model based automatic control of such processes aimed at bending of the heated and plasticized details.*

Key words: unilateral heating, modeling, wood details, plasticizing, bending, specific energy consumption

SAŽETAK • *Autori u radu daju prijedlog metodologije za matematičko modeliranje i istraživanje dvaju međusobno povezanih problema: raspodjele temperature u smjeru debljine ravnih drvenih elemenata izloženih jednostranom zagrijavanju i potrošnje energije tog procesa. Za realizaciju metodologije izrađen je jednodimenzionalni matematički model i riješen za linearni tok topline pri provođenju topline u ravnim drvenim elementima za vrijeme njihova proizvoljnoga jednostranog zagrijavanja, s početnim i rubnim uvjetima koji se mogu susresti u praksi. Na temelju integracije rješenja modela, predložena je numerička metoda za izračunavanje specifične potrošnje energije (za 1 m^2) i specifičnoga toplinskog toka potrebnoga za zagrijavanje elemenata i za nadoknađivanje emisije topline u okolinu za vrijeme jednostranoga grijanja radi plasticiranja i lakšeg savijanja drva.*

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U radu su prikazana rješenja modela koja se odnose na nestacionarnu raspodjelu temperature smrekovih elemenata debljine 6, 8 i 10 mm po debljini i na nestacionarnu promjenu specifične potrošnje energije i specifičnoga toplinskog toka za vrijeme jednostranog zagrijavanja na temperature električno grijane metalne trake koje su iznosile 100, 120 i 140 °C. Elementi su zagrijavani radi plasticiranja i savijanja drvenih elemenata u proizvodnji vanjskih zakrivljenih dijelova korpusa žičanih glazbenih instrumenata. Dobiveni se rezultati mogu koristiti za tehnološke i energetske izračune i za analizu procesa jednostranog zagrijavanja drvenih elemenata u različitim rubnim uvjetima, kao i za softver za modeliranje sustava koji se temelji na automatskom upravljanju procesima usmjerenima na savijanje zagrijanih i plastificiranih drvenih elemenata.

Ključne riječi: *jednostrano grijanje, modeliranje, drveni elementi, plastificiranje, savijanje, specifična potrošnja energije*

1 INTRODUCTION
1. UVOD

An important mandatory component of technologies for the production of curved wood details in furniture industry and others is their plasticizing up to the stage that allows for their faultless bending (Angelski, 2010). The current applications of such technologies differ mainly by the method selected for plasticizing wood (by heating using water steam, hot water, heated metal curved surfaces, or high frequency electric current).

The method for plasticizing wood details before their bending is chosen primarily based on economic reasons and according to the values of the relationship *R/h*, where *R* is the radius of bending curvature of the concave side of the details, and *h* is the size of the cross section of details in the direction coinciding with the bending plane.

During the bending of details with large radii of curvature at *R/h* > 20, the wood need not have a large degree of plasticity. In this case, from technological and economical standpoints, it is most advantageous to plasticize the wood using unilateral heating at relatively low wood moisture content in the range from 12 % to 20 % (Rowell and Konkol, 1987; Rice and Lucas, 2003). Plasticizing with such, relatively low, wood moisture content ensures considerable decreasing of the duration of the drying of the bent details aimed at providing lasting stabilization of their form.
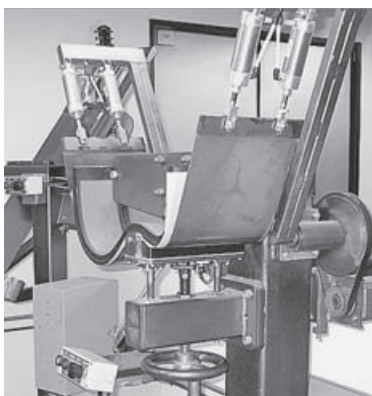

Figure 1 Equipment with electrically heated band for unilateral heating and bending of flat wood details in the production of outside curved parts for bodies of string musical instruments
Slika 1. Oprema s električno grijanom trakom za jednostrano zagrijavanje i savijanje ravnih drvenih elemenata u proizvodnji vanjskih zakrivljenih dijelova korpusa gudaćih instrumenata

Unilateral heating of wood details is most often carried out in the specific equipment used for bending. For such heating of details with thicknesses between 10 and 25 mm, hot hydraulic presses with appropriately bent surfaces are usually used. Curved details for the back parts of chairs are produced, for example, by this method of plasticizing. These details have a relatively small thickness, a large radius of curvature and *R/h ratio* = 20 - 25.

In the production of curved outside parts for the bodies of string musical instruments (violins, guitars, mandolins, violoncellos), details with thickness between 5 mm and 10 mm and with a 15 % moisture content are subjected to bending. The technology for plasticizing of such details usually uses equipment with metal band, electrically heated up to the temperature in the range from 100 ºC to 150 ºC (Figure 1).

It is known that the degree of plasticizing and deformability of the wood depends considerably on the temperature distribution in the details' volume at the moment of bending. The non-stationary temperature distribution in the details and the specific energy consumption during their unilateral heating, aimed at the details' plasticizing before bending, depends on many factors: wood species, thickness and moisture content of the details, temperatures of the heating medium (body) and of the surrounding air, desired degree of plasticizing, radius of the bending, etc. (Chudinov, 1968; Shubin, 1990; Požgaj *et al*., 1997; Taylor, 2001; Trebula and Klement, 2002; Deliiski, 2003, 2013b; Videlov, 2003; Pervan, 2009; Angelski, 2010; Deliiski and Dzurenda, 2010; Gaff and Prokein, 2011).

In the specialized literature, there is little data available about the temperature distribution in wood details subjected to multilateral heating with water steam before bending (Angelski, 2010) and there is no information at all about the temperature distribution in details during their unilateral heating. Information about the energy consumption needed for the heating of wood details during their unilateral heating was provided only by the authors (Deliiski *et al*., 2014). In the cited publication, an approach for the calculation of the specific energy consumption needed only for the wood heating in kWh·m^{-3} was presented and there is no information about the energy needed for the covering of the heat emission from the non-heated side of wood details.

That is why the modeling and energy consumption of the processes of unilateral heating of wood details, as well as the multiparameter study of these pro-

cesses, are of considerable scientific and practical interest. For the practical needs, it is more convenient to determine the total energy consumption during unilateral heating of flat details in kWh·m⁻² instead in kWh·m⁻³.

The aim of the present paper is to suggest a methodology for mathematical modeling and research of two mutually connected problems: the temperature distribution in flat wood details subjected to unilateral heating and the energy consumption of this process.

The paper presents the development and solution of 1-dimentional mathematical model of the transient linear heat conduction in flat wood details during their unilateral heating at arbitrary initial and boundary conditions encountered in the practice. Based on the integration of the model's solutions, for the first time a numerical approach will be suggested for the computation of the specific (for 1 m²) energy consumption and the specific heat flux needed for the heating of the details and for the covering of their heat emission during unilateral heating aimed at wood plasticizing and bending.

The paper also presents the results of simulation research of the non-stationary temperature distribution along the thickness of spruce details with h = 6, 8 and 10 mm and the non-stationary change in the specific energy consumption and in the heat flux during unilateral heating at temperatures of the electrically heated metal band t_m = 100, 120, and 140 ºC aimed at plasticizing and bending of the details in the production of outside curved parts for the bodies of string musical instruments.

2 MATERIAL AND METHODS
2. MATERIJAL I METODE

2.1 Modeling of heat distribution in flat wood details during their unilateral heating
2.1. Modeliranje raspodjele topline u ravnim drvenim elementima tijekom jednostranog zagrijavanja

When the width and length of wood details exceed their thickness by at least 3 and 5 times, respectively, then the change in the temperature only along the thickness of the details in the center of their flat side during unilateral heating (i.e. along the coordinate x, which coincides with the details thickness h) can be calculated with the help of the following linear 1D mathematical model (Deliiski, 2003, 2011, 2013b):

$$\frac{\partial T(x,\tau)}{\partial \tau} = a_w(T,u,u_{fsp},\rho_b,S_v) \cdot \frac{\partial^2 T(x,\tau)}{\partial x^2} \quad (1)$$

with an initial condition

$$T(x, 0) = T_0 \quad (2)$$

and following boundary conditions:
• from the side of the details heating – at prescribed surface temperature, which is equal to the temperature of the metal heating band T_m:

$$T(0, \tau) = T_m(\tau). \quad (3)$$

• from the opposite non-heated side of the details – at convective heat exchange between the details surface and the surrounding air environment

$$\frac{\partial T(X,\tau)}{\partial x} = -\frac{\alpha}{\lambda_{ws}(T,u,\rho_b)} \cdot [T_{ws}(\tau) - T_a(\tau)] \quad (4)$$

For practical use of eqs. (1) and (4), it is necessary to have mathematical descriptions of wood temperature conductivity, a_w, of wood thermal conductivity, λ_w, and of the heat transfer coefficient between the details surface at their non-heated side and the surrounding air, α. For this purpose, the mathematical description of a_w and λ_w of non-frozen wood given in (Deliiski, 2003, 2013a; Deliiski et al., 2015) can be used.

The heat transfer coefficient, α can be calculated with the help of the following equation, which is valid for the cases of cooling or heating of horizontally situated rectangular surfaces in atmospheric conditions of free convection (Chudinov, 1968; Telegin et al., 2002):

$$\alpha = 3.256 \cdot [T_{ws}(\tau) - T_a(\tau)]^{0.25} \quad (5)$$

According to eq. (3), the temperature at the details surface, which is in contact with the heating metal band, is accepted to be equal to its temperature T_m due to the extremely high coefficient of heat transfer between the band and the wood during their very close contact.

2.2 Modeling of the total specific energy consumption for unilateral heating of wood details
2.2. Modeliranje ukupne specifične potrošnje energije za jednostrano zagrijavanje drvenih elemenata

The total specific energy consumption for unilateral heating of wood details, q_{total}, consists of two components:
• energy needed for heating of the wood itself, q_w;
• energy needed for the covering of the heat emission from the non-heated side of wood details, q_e.

This means that the energy q_{total} can be calculated according to the following equation:

$$q_{total} = q_w + q_e. \quad (6)$$

Analogously, the total heat flux needed for unilateral heating of wood details, $dq_{total}/d\tau$, can be calculated according to the equation

$$\frac{dq_{total}}{d\tau} = \frac{dq_w}{d\tau} + \frac{dq_e}{d\tau}. \quad (7)$$

2.2.1 Modeling of the specific energy consumption for heating of the wood itself
2.2.1. Modeliranje specifične potrošnje energije samo za zagrijavanje drva

It is known that the specific energy consumption for heating 1 m³ of solid wood with an initial mass temperature T_0 to a given average mass temperature T_{avg} is determined using the equation (Deliiski, 2013b; Deliiski et al., 2014)

$$q = \frac{c_w(T,u) \cdot \rho_w(\rho_b,u,u_{fsp})}{3.6 \cdot 10^6} \cdot (T_{avg} - T_0). \quad (8)$$

After multiplying the right part of eq. (8) by the detail thickness h, the following equation is obtained for the determination of the specific mass energy con-

sumption needed for heating 1 m² of wood details subjected to unilateral heating, q_w:

$$q_w = \frac{c_w(T,u) \cdot \rho_w(\rho_b, u, u_{fsp}) \cdot h}{3.6 \cdot 10^6} \cdot \left(T_{avg} - T_0\right), \qquad (9)$$

where

$$T_{avg} = \frac{1}{h} \int_{(h)} T(x,\tau) dx \qquad (10)$$

and according to Chudinov (1968) and Deliiski (2013b)

$$\rho_w = \rho_b \cdot \frac{1+u}{1 - \frac{S_v}{100} \cdot \left(u_{fsp}^{293.15} - u\right)} \quad @ \quad u \le u_{fsp}. \qquad (11)$$

The multiplier $3.6 \cdot 10^6$ in the denominator of eq. (9) ensures that the values of q_w are obtained in kWh·m⁻², instead of in J·m⁻².

The change in q_w during the time $\Delta\tau$, i.e. the heat flux needed for heating 1 m² of wood details subjected to unilateral heating, $dq_w/d\tau$ (in kW·m⁻²), can be calculated according to equation

$$\frac{dq_w}{d\tau} \approx \frac{3600 \cdot \Delta q_w}{\Delta\tau}. \qquad (12)$$

For practical use of eqs. (9) and (12), it is necessary to have a mathematical descriptions of the specific heat capacity of the non-frozen wood, c_w. Such descriptions are given in (Deliiski, 2003, 2011, 2013b; Deliiski *et al.*, 2015; Radmanović *et al.*, 2014; Deliiski and Dzurenda, 2010).

2.2.2 Modeling of specific energy consumption for covering of the heat emission from the non-heated side of wood details
2.2.2. Modeliranje specifične potrošnje energije za nadoknađivanje emisije topline nezagrijavane strane drvenih elemenata

The change in the specific energy consumption q_e (for 1 m² of the details surface), which is required for the covering of the heat emission from the non-heated side of wood details into the surrounding air environment during the time $\Delta\tau$, can be calculated according to the following equation (Deliiski, 2003; Deliiski *et. al.* 2014a):

$$\Delta q_e = \frac{\alpha(\tau)\Delta\tau}{3.6 \cdot 10^6} \cdot \left[T_{ws}(\tau) - T_a(\tau)\right]. \qquad (13)$$

The specific energy consumption required for the covering of the heat emission from 1 m² surface of the details during their unilateral heating with duration $\tau_p = N \cdot \Delta\tau$ is equal to

$$q_e = \sum_{i=1}^{N} \Delta q_{ei}. \qquad (14)$$

The change in q_e during the time $\Delta\tau$, i.e. the heat flux required for the covering of the heat emission from the non-heated side of 1 m² of wood details subjected to unilateral heating, $dq_e/d\tau$ (in kW·m⁻²), can be calculated according to equation (Deliiski *et al.*, 2016b)

$$\frac{dq_e}{d\tau} \approx \frac{3600 \Delta q_e}{\Delta\tau}. \qquad (15)$$

2.3 Transformation of the model to a form suitable for programming
2.3. Transformacija modela u oblik pogodan za programiranje

The following system of equations was derived by passing to final increases in equations (1) to (5) with

the use of the same approach, as well as by the explicit form of the finite-difference method described in Deliiski (2003, 2011, 2013b; Deliiski and Dzurenda, 2010):

$$T_i^{n+1} = T_i^n + \frac{a_w \Delta\tau}{\Delta x^2} \cdot \left(T_{i-1}^n + T_{i+1}^n - 2T_i^n\right), \qquad (16)$$

$$T_i^0 = T_0 \quad @ \quad 1 \le i \le M = 9, \qquad (17)$$

$$T_1^{n+1} = T_m^{n+1}, \qquad (18)$$

$$T_9^{n+1} = \frac{T_8^n + \frac{\alpha^n \cdot \Delta x \cdot T_a^n}{\lambda_{ws}}}{1 + \frac{\alpha^n \cdot \Delta x}{\lambda_{ws}}}, \qquad (19)$$

$$\alpha^n = 3.256 \cdot \left[T_9^n - T_a^n\right]^{0.25}, \qquad (20)$$

where

$$\Delta x = \frac{h}{M-1}, \qquad (21)$$

$$M = \frac{h}{\Delta x} + 1, \qquad (22)$$

$$\Delta\tau \le \frac{\Delta x^2}{a}. \qquad (23)$$

The equations (9) and (10) for the computation of the specific energy consumption needed for unilateral heating of wood details results in the following form suitable for programming:

$$q_w^{n+1} = \frac{\rho_w \cdot c_w \cdot h}{3.6 \cdot 10^6} \cdot \left(T_{avg}^{n+1} - T_0\right), \qquad (24)$$

where

$$T_{avg}^{n+1} = \frac{1}{h} \int_{(h)} T[x, (n+1)\Delta\tau] dx. \qquad (25)$$

The highest precision for the solution of eq. (25) is ensured by the use of the Simpson's method instead of the well-known but less accurate trapezoidal method and the method of Gregori's (Deliiski, 2003). According to this method, eq. (25) obtains the following discrete analogue (Dorn and McCracken, 1972):

$$T_{avg}^{n+1} = \frac{\Delta x}{3}\left(\begin{array}{l}T_1^{n+1} + 4T_2^{n+1} + 2T_3^{n+1} + 4T_4^{n+1} + \\ 2T_5^{n+1} + 4T_6^{n+1} + 2T_7^{n+1} + 4T_8^{n+1} + T_9^{n+1}\end{array}\right). \quad (26)$$

Equation (12) for the computation of the specific heat flux needed for unilateral heating of wood details obtains the following form suitable for programming:

$$\frac{dq_w^{n+1}}{d\tau} \approx \frac{3600 \Delta q_w^{n+1}}{\Delta\tau}, \qquad (27)$$

where

$$\Delta q_w^{n+1} = q_w^{n+1} - q_w^n, \qquad (28)$$

$$q_w^n = \frac{\rho_w \cdot c_w \cdot h}{3.6 \cdot 10^6} \cdot \left(T_{avg}^n - T_0\right), \qquad (29)$$

$$T_{avg}^n = \frac{\Delta x}{3}\left(\begin{array}{l}T_1^n + 4T_2^n + 2T_3^n + 4T_4^n + \\ 2T_5^n + 4T_6^n + 2T_7^n + 4T_8^n + T_9^n\end{array}\right). \quad (30)$$

The change in the specific energy consumption q_e (for 1 m² of the details surface), required for the covering of the heat emission from the non-heated side of wood details into the surrounding air environment during the time $\Delta\tau$, can be calculated according to the following equation (Deliiski 2003, Deliiski *et al.*, 2016b):

$$\Delta q_e^{n+1} = \frac{\alpha^n(\tau)\Delta\tau}{3.6 \cdot 10^6} \cdot \left[T_9^n(\tau) - T_a^n(\tau) \right]. \qquad (31)$$

Equation (15) for the computation of the specific heat flux needed for the covering of the heat emission from the non-heated side of wood details obtains the following form suitable for programming:

$$\frac{dq_e^{n+1}}{d\tau} \approx \frac{3600 \cdot \Delta q_e^{n+1}}{\Delta\tau}. \qquad (32)$$

The step on the time coordinate $\Delta\tau$ is determined by keeping the condition for stability of the solution of 1D models by using the explicit form of the finite difference method (Deliiski, 2003, 2011):

$$\Delta\tau \le \frac{\Delta x^2}{a_w}. \qquad (33)$$

3 RESULTS AND DISCUSSION
3. REZULTATI I RASPRAVA

For the numerical solution of the above presented mathematical model, a software program was prepared in FORTRAN in the calculation environment of Visual Fortran Professional.

With the help of the program, as examples, computations were made for the determination of the 1D non-stationary change of t, t_{avg}, q_w, q_e, q_{total}, $dq_w/d\tau$, $dq_e/d\tau$, and $dq_{total}/d\tau$ for non-frozen spruce (*Picea Abies Karst*) details with $h = 6$ mm, $h = 8$ mm, $h = 10$ mm, $t_0 = 20$ ºC, $u = 0.15$ kg·kg⁻¹, $\rho_b = 380$ kg·m⁻³, and $S_v = 11.4$ % during their 10 min unilateral heating at $t_m = 100$ ºC, $t_m = 120$ ºC, $t_m = 140$ ºC, and at $t_a = 20$ ºC.

The computations were carried out with average values of the spruce temperature conductivity cross-sectional to the fibers, a_w, and values of the spruce thermal conductivity cross-sectional to the fibers, $\lambda_w = \lambda_{ws}$, which have been obtained using the mathematical descriptions of a_w and λ_w depending on t, u, and u_{fsp} of the wood species (Deliiski, 2003, 2013a, Deliiski *et. al.*, 2015). The calculated values of a_w and λ_w with the help of these mathematical descriptions for spruce wood with $u = 0.15$ kg·kg⁻¹ and $u_{fsp}^{293.15} = 0.32$ kg·kg⁻¹ (Nikolov and Videlov, 1987; Deliiski and Dzurenda, 2010) in the temperature ranges from 20 ºC to 100 ºC, from 20 ºC to 120 ºC, and from 20 ºC to 140 ºC, are shown in Table 1.

The linear dependences of a_w and of $\lambda_{ws} = \lambda_w$ on t (Deliiski, 2003, 2013a, 2013b; Deliiski *et al.*, 2015) allow for the solution of the mathematical model using the average arithmetic values of a_w and $\lambda_{ws} = \lambda_w$ in respective temperature ranges (Table 1) for the determination of the temperature distribution along the details thickness during unilateral heating of the details.

Simultaneously with the computation of the non-stationary distribution of t along the details thickness, calculations of t_{avg}, t_{ws}, α, q_w, q_e, q_{total}, $dq_w/d\tau$, $dq_e/d\tau$, and $dq_{total}/d\tau$ were carried out, using the value of the density $\rho_w = 445.6$ kg·m⁻³ and average arithmetic values of the specific heat capacity c_w given in the last row of Table 1 in respective temperature ranges. This value of ρ_w is calculated according to eq. (11) for spruce wood with $u = 0.15$ kg·kg⁻¹, $u_{fsp}^{293.15} = 0.32$ kg·kg⁻¹, $\rho_b = 380$ kg·m⁻³, and $S_v = 11.4$ % (Nikolov and Videlov, 1987). The values of c_w were calculated according to the mathematical description of the specific heat capacity of the non-frozen wood in the hygroscopic range given in (Deliiski, 2003, 2011, 2013b). Because of the almost linear dependence of c_w on t, the average arithmetic values of c_w for the respective temperature ranges were used during the solution of eqs. (24) and (29).

The left part of Figure 2 presents the temperature change calculated by the model in 4 characteristic points, equidistant from one another, along the thickness of the detail with $h = 8$ mm during its unilateral heating at $t_m = 140$ ºC. The coordinates of these points are shown in the legend of the figure. The right part of this figure shows the change in t_{avg} of the details during their heating at $t_m = 140$ ºC, depending on h.

Figure 3 presents the change in the surface temperature at the non-heated side of the details, t_{ws}, and in the heat transfer coefficient, α, during unilateral heating at $t_m = 140$ ºC, depending on h.

Figures 4, 5, and 6 present the calculated change of q_w, q_e, and q_{total} during unilateral heating at $t_m = 100$ ºC and at $t_m = 140$ ºC of the spruce details with studied thicknesses.

Figures 7, 8, and 9 present the calculated change of $dq_w/d\tau$, $dq_e/d\tau$, and $dq_{total}/d\tau$ during unilateral heating at $t_m = 100$ ºC and at $t_m = 140$ ºC of the spruce details with studied thicknesses.

Table 1 Change in a_w, $\lambda_w = \lambda_{ws}$, and c_w of spruce wood with $u = 0.15$ kg.kg⁻¹, depending on t
Tablica 1. Promjena vrijednosti a_w, $\lambda_w = \lambda_{ws}$, i c_w za smrekovinu sa sadržajem vode $u = 0,15$ kg·kg⁻¹, ovisno o temperaturi t

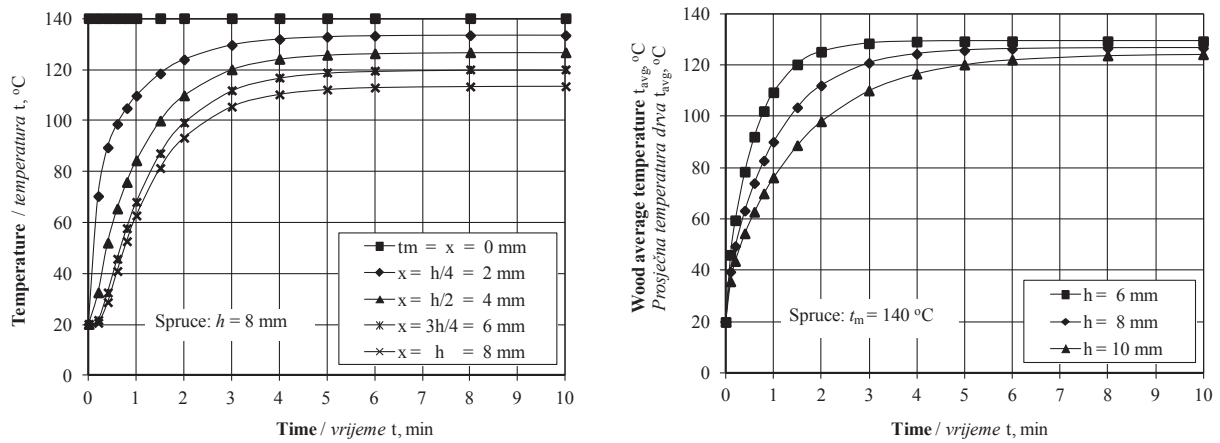| Wood parameter *Svojstvo drva*	Temperature *t*, ºC *Temperatura t, ºC*				Average arithmetic values of λ_w, a_w, and c_w for temperature ranges: *Prosječne vrijednosti λ_w, a_w, i c_w za raspon temperatura*		
	20	60	100	140	t =20÷100 ºC	t =20÷120 ºC	t=20÷140 ºC
$\lambda_{ws} = \lambda_w$, W·m⁻¹·K⁻¹	0.2341	0.2664	0.2987	0.3311	0.2664	0.2745	0.2826
$a_w \cdot 10^7$, m²·s⁻¹	2.5799	2.7412	2.8818	3.0052	2.7309	2.7627	2.7926
c_w, J·kg⁻¹·K⁻¹	2036	2181	2326	2472	2181	2218	2254

Figure 2 Change in *t* along the detail thickness of *h* = 8 mm (left) and in t_{avg} (right) of spruce details with t_0 = 20 ºC, *u* = 0.15 kg·kg⁻¹, *h* = 6 mm, *h* = 8 mm, and *h* = 10 mm during their unilateral heating at t_m = 140 ºC and at t_a = 20 ºC
Slika 2. Promjena temperature *t* u smjeru debljine elementa *h* = 8 mm (lijevo) i prosječne temperature drva t_{avg} (desno) smrekovih elemenata pri t_0 = 20 ºC, *u* = 0,15 kg·kg⁻¹, *h* = 6 mm, *h* = 8 mm i *h* = 10 mm tijekom njihova jednostranog zagrijavanja pri t_m = 140 ºC i t_a = 20 ºC

Figure 3 Change in t_{ws} (left) and in α (right) of spruce details with t_0 = 20 ºC, *u* = 0.15 kg·kg⁻¹ during their unilateral heating at t_m = 140 ºC and at t_a = 20 ºC, depending on *h*
Slika 3. Promjena temperature t_{ws} (lijevo) i koeficijenta prijenosa topline α (desno) smrekovih elemenata pri t_0 = 20 ºC, *u* = 0,15 kg·kg⁻¹ tijekom njihova jednostranog zagrijavanja pri t_m = 140 ºC i t_a = 20 ºC, ovisno o debljini elemenata *h*

Figure 4 Change in q_w of spruce details with t_0 = 20 ºC and *u* = 0.15 kg·kg⁻¹ during their unilateral heating at t_m = 100 ºC (left) and t_m = 140 ºC (right), depending on *h*
Slika 4. Promjena specifične energije zagrijavanja q_w smrekovih elemenata pri t_0 = 20 ºC i *u* = 0,15 kg·kg⁻¹ tijekom njihova zagrijavanja pri t_m = 100 ºC (lijevo) i t_m = 140 ºC (desno), ovisno o debljini elemenata *h*
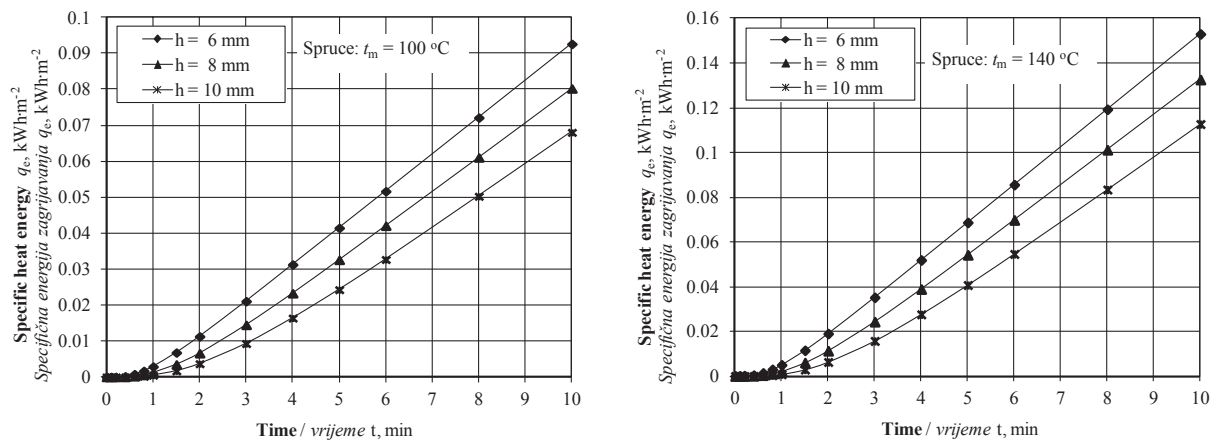
Figure 5 Change in q_e of spruce details with $t_0 = 20$ ºC and $u = 0.15$ kg·kg⁻¹ during their unilateral heating at $t_m = 100$ ºC (left) and $t_m = 140$ ºC (right), depending on h
Slika 5. Promjena specifične energije zagrijavanja q_e smrekovih elemenata pri $t_0 = 20$ ºC i $u = 0.15$ kg·kg⁻¹ tijekom njihova jednostranog zagrijavanja pri $t_m = 100$ ºC (lijevo) i $t_m = 140$ ºC (desno), ovisno o debljini elemenata h

The obtained results show that during unilateral heating of the details, the change of all studied parameters of the process takes place according to complex curves.

By increasing the heating time, the curves of t gradually approach asymptotically their largest values, decreasingly dependent on the remoteness of the characteristic points from the heated surface of the details. Analogously, the curves of the change in t_{avg}, α, q_w, and $dq_e/d\tau$ approach asymptotically their largest values, increasingly dependent on t_m and decreasingly dependent on h. The largest values of t, t_{avg}, α, q_w, and $dq_e/d\tau$ are achieved when a stationary temperature distribution occurs along the details thickness.

The stationary temperature distribution along the thickness of the studied spruce details (with precision of up to –0.2 ºC) occurs upon reaching the following temperature at their non-heated side:
• for details with $h = 6$ mm: 86.0 ºC after heating duration of 3.5 min at $t_m = 100$ ºC, 102.7 ºC after heating duration of 3.8 min at $t_m = 120$ ºC, 118.5 ºC after heating duration of 3.9 min at $t_m = 140$ ºC;

• for details with $h = 8$ mm: 82.7 ºC after heating duration of 6.4 min at $t_m = 100$ ºC, 98.0 ºC after heating duration of 6.7 min at $t_m = 120$ ºC, 113.3 ºC after heating duration of 6.8 min at $t_m = 140$ ºC;
• for details with $h = 10$ mm: 79.7 ºC after heating duration of 9.7 min at $t_m = 100$ ºC, 94.2 ºC after heating duration of 10.2 min at $t_m = 120$ ºC, 108.7 ºC after heating duration of 10.4 min at $t_m = 140$ ºC.

At $t_m = 100$ ºC, the most slowly changing temperature of the details surface, t_{ws}, which is in contact with the outside air environment, reaches temperatures of 50 ºC, 60 ºC, 70 ºC and 80 ºC, which are necessary for the start of the bending of the details with different radii R after a duration of unilateral heating, equal, respectively, to: 0.6 min, 0.8 min, 1.0 min and 1.8 min at $h = 6$ mm; 1.1 min, 1.5 min, 2.1 min and 3.9 min at $h = 8$ mm; 1.7 min, 2.4 min, 3.6 min and ≈10.0 min at $h = 10$ mm.

At $t_m = 120$ ºC, the temperature t_{ws} reaches temperatures of 50 ºC, 60 ºC, 70 ºC 80 ºC and 90 ºC, after a duration of one sided heating, equal, respectively, to: 0.5 min, 0.6 min, 0.8 min, 1.0 min and 1.min at $h = 6$
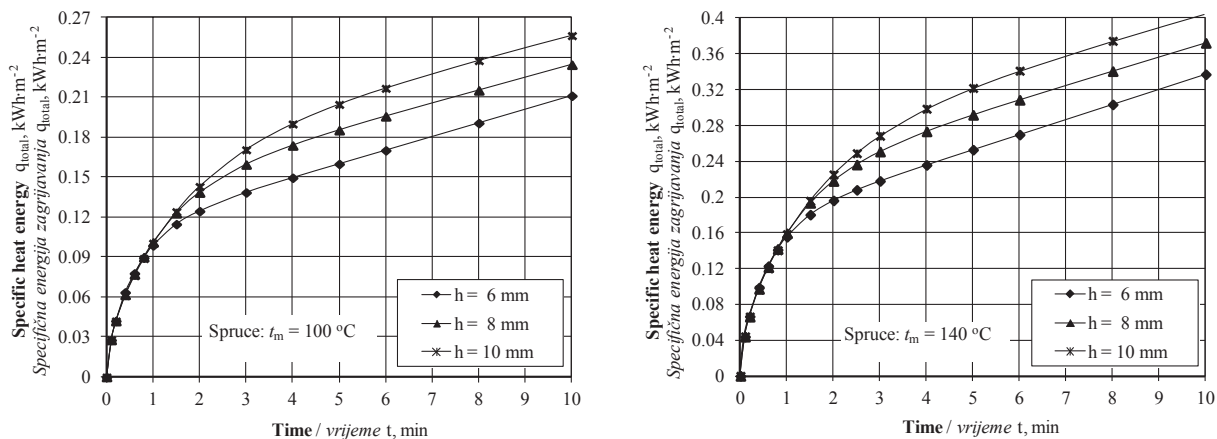

Figure 6 Change in q_{total} of spruce details with $t_0 = 20$ ºC and $u = 0.15$ kg·kg⁻¹ during their unilateral heating at $t_m = 100$ ºC (left) and $t_m = 140$ ºC (right), depending on h
Slika 6. Promjena specifične energije zagrijavanja q_{total} smrekovih elemenata pri $t_0 = 20$ ºC i $u = 0.15$ kg·kg⁻¹ tijekom njihova jednostranog zagrijavanja pri $t_m = 100$ ºC (lijevo) i $t_m = 140$ ºC (desno), ovisno o debljini elemenata h

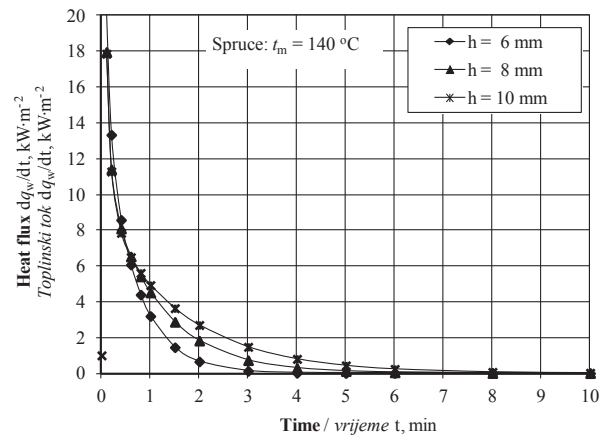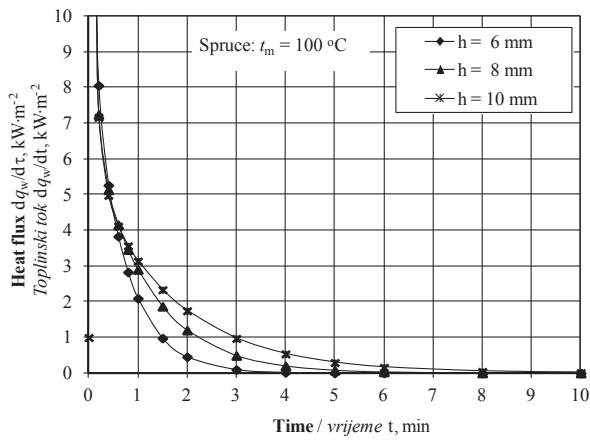**Figure 7** Change in $dq_w/d\tau$ of spruce details with $t_0 = 20$ ºC and $u = 0.15$ kg·kg⁻¹ during their unilateral heating at $t_m = 100$ ºC (left) and $t_m = 140$ ºC (right), depending on h

Slika 7. Promjena toplinskog toka $dq_w/d\tau$ smrekovih elemenata pri $t_0 = 20$ ºC i $u = 0,15$ kg·kg⁻¹ tijekom njihova jednostranog zagrijavanja pri $t_m = 100$ ºC (lijevo) i $t_m = 140$ ºC (desno), ovisno o debljini elemenata h
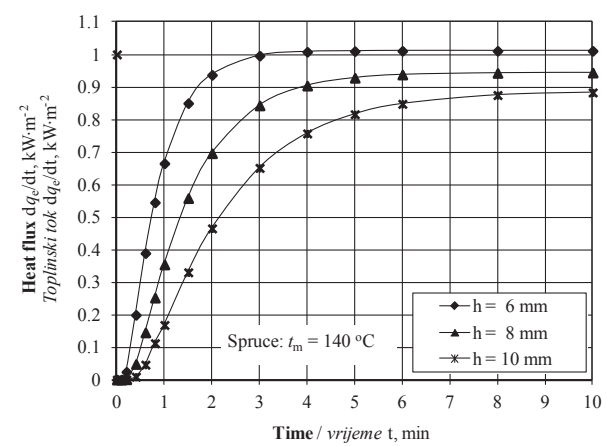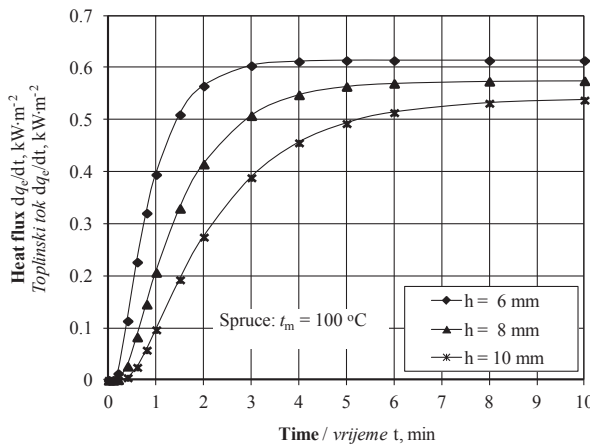

Figure 8 Change in $dq_e/d\tau$ of spruce details with $t_0 = 20$ ºC and $u = 0.15$ kg·kg⁻¹ during their unilateral heating at $t_m = 100$ ºC (left) and $t_m = 140$ ºC (right), depending on h

Slika 8. Promjena toplinskog toka $dq_e/d\tau$ smrekovih elemenata pri $t_0 = 20$ ºC i $u = 0,15$ kg·kg⁻¹ tijekom njihova jednostranog zagrijavanja pri $t_m = 100$ ºC (lijevo) i $t_m = 140$ ºC (desno), ovisno o debljini elemenata h

Figure 9 Change in $dq_{total}/d\tau$ of spruce details with $t_0 = 20$ ºC and $u = 0.15$ kg·kg⁻¹ during their unilateral heating at $t_m = 100$ ºC (left) and $t_m = 140$ ºC (right), depending on h

Slika 9. Promjena toplinskog toka $dq_{total}/d\tau$ smrekovih elemenata pri $t_0 = 20$ ºC i $u = 0,15$ kg·kg⁻¹ tijekom njihova jednostranog zagrijavanja pri $t_m = 100$ ºC (lijevo) i $t_m = 140$ ºC (desno), ovisno o debljini elemenata h

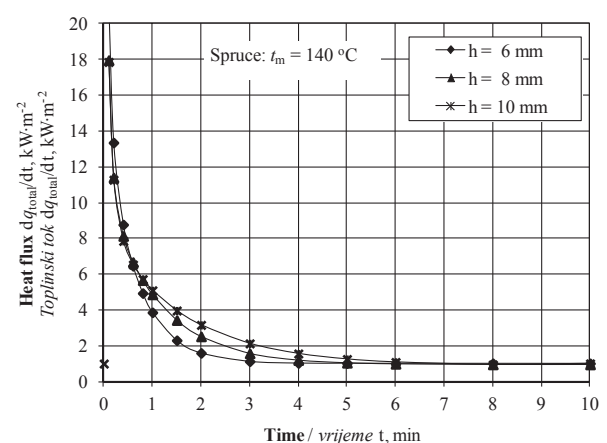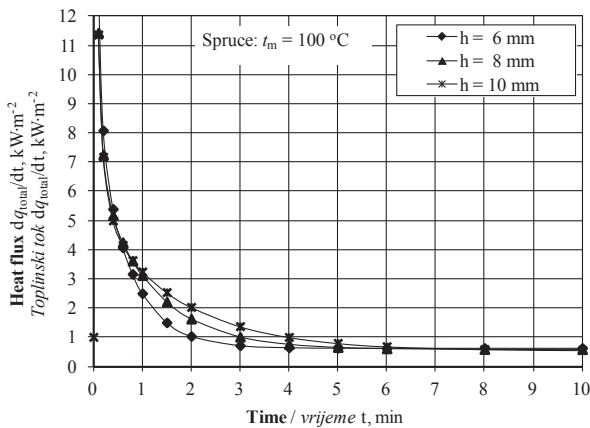mm; 0.9 min, 1.2 min, 1.5 min, 2.0 min and 2.9 min at $h = 8$ mm; 1.4 min, 1.8 min, 2.4 min, 3.3 min and 5.3 min at $h = 10$ mm.

At $t_m = 140$ ºC, the temperature t_{ws} reaches temperatures of 50 ºC, 60 ºC, 70 ºC 80 ºC, 90 ºC and 100 ºC, after a duration of heating, equal, respectively, to: 0.4 min, 0.5 min, 0.6 min, 0.8 min, 1.0 min and 1.3 min at $h = 6$ mm; 0.8 min, 1.0 min, 1.2 min, 1.5 min, 1.9 min and 2.5 min at $h = 8$ mm; 1.2 min, 1.5 min, 1.9 min, 2.5 min, 3.1 min and 4.3 min at $h = 10$ mm.

By increasing the heating time, q_e and q_{total} increase according to curvilinear dependences, which change into linear after the reaching of stationary distribution of t along the details thickness. The slopes of the linear sections of dependences $q_e = f(\tau)$ and $q_{total} = f(\tau)$ are proportional to t_m.

The values of q_{total} are increasingly dependent on t_m and decreasingly dependent on h. For example, after 10 min duration of unilateral heating, q_{total} reaches the following values:
– for details with $h = 6$ mm: $q_{total} = 0.2115$ kWh·m⁻² at $t_m = 100$ ºC, $q_{total} = 0.2730$ kWh·m⁻² at $t_m = 120$ ºC, and $q_{total} = 0.3372$ kWh·m⁻² at $t_m = 140$ ºC;
– for details with $h = 8$ mm: $q_{total} = 0.2350$ kWh·m⁻² at $t_m = 100$ ºC, $q_{total} = 0.3022$ kWh·m⁻² at $t_m = 120$ ºC, and $q_{total} = 0.3723$ kWh·m⁻² at $t_m = 140$ ºC;
– for details with $h = 10$ mm: $q_{total} = 0.2569$ kWh·m⁻² at $t_m = 100$ ºC, $q_{total} = 0.3260$ kWh·m⁻² at $t_m = 120$ ºC, and $q_{total} = 0.4053$ kWh·m⁻² at $t_m = 140$ ºC.

The change of $dq_w/d\tau$ and $dq_{total}/d\tau$ during the heating time also takes place according to complex curves. In the beginning of heating, these parameters increase rapidly to values, which are beyond the grading of the Y-axis in Fig. 7 and Fig. 9. This rapid increase of $dq_w/d\tau$ and $dq_{total}/d\tau$ corresponds to the steep initial sections of dependencies $q_w = f(\tau)$ and $q_{total} = f(\tau)$. After reaching their maximum values, the heat fluxes $dq_w/d\tau$ and $dq_{total}/d\tau$ begin to decrease – initially very sharply and then more smoothly. When a stationary distribution of t along the details thickness occurs, $dq_w/d\tau$ becomes equal to 0. Then $dq_{total}/d\tau$ approaches asymptotically its lowest values, increasingly dependent on t_m and decreasingly dependent on h. For example, these lowest values of $dq_{total}/d\tau$ for $h = 10$ mm are equal to 0.552 kW·m⁻² at $t_m = 100$ ºC, to 0.726 kW·m⁻² at $t_m = 120$ ºC, and to 0.905 kW·m⁻² at $t_m = 140$ ºC.

Using the values of q_{total} and $dq_{total}/d\tau$, the minimum necessary power of the heating metal band (refer to Fig. 1) can be determined depending on the desired duration of unilateral heating of details at given values for t_m, h and R/h.

4 CONCLUSIONS
4. ZAKLJUČAK

This paper presents a methodology for mathematical modeling and research of two mutually connected problems: the temperature distribution in flat wood details subjected to unilateral heating and the energy consumption of this process.

The paper describes the development and solution of a 1-dimensional mathematical model of the transient linear heat conduction in flat wood details during their unilateral heating at arbitrary initial and boundary conditions encountered in the practice. Heat distribution along the thickness of wood details from any wood species subjected to such heating is described by the 1D partial differential equation of heat conduction. For the solution of the model, an explicit form of the finite-difference method is used.

Based on the integration of the model's solutions, for the first time a numerical approach is suggested for the computation of the specific energy consumption (for 1 m²) and the specific heat flux needed for the heating of the details and for the covering of their heat emission during unilateral heating aimed at wood plasticizing and bending.

For the numerical solution of the mathematical model, a software program was prepared in FORTRAN in the calculation environment of Visual Fortran Professional.

The paper shows and analyzes diagrams of a 1D non-stationary and stationary distribution of the temperature along the thickness of flat spruce details subjected to unilateral heating in order to be plasticized before their bending for the production of outside parts for the bodies of string musical instruments. It also shows and analyzes the change of the total specific energy consumption, q_{total}, and its components, and of the total specific heat flux, $dq_{total}/d\tau$, and its components needed for unilateral heating of details. All diagrams are drawn using the results calculated by the model.

The 1D distribution of the temperature and the change of q_{total}, $dq_{total}/d\tau$ and of their components for flat spruce details with thickness of 6 mm, 8 mm and 10 mm, initial wood temperature of 20 ºC, and moisture content of 0.15 kg.kg⁻¹ during their unilateral heating of 10 min at temperatures of the heating metal body of $t_m = 100$ ºC, $t_m = 120$ ºC, $t_m = 140$ ºC, and temperature of the air near the non-heated side of the details of $t_a = 20$ ºC have been calculated, visualized and analyzed using the model.

The suggested methodology and algorithm for its realization, as well as the results obtained from the computer solutions of the model could be used for the following purposes:
– Visualization and technological analysis of the temperature change along the thickness of details from different wood species with different thickness and moisture content during their unilateral heating before bending accomplished by a heating body with different temperatures;
– Determination of the time of details heating, which is necessary for achieving the minimum required plasticity of the details before their bending with a specified radius and R/h ratio;
– Computation of the energy consumption of the details at each moment of their unilateral heating;
– Determination of the necessary power of the heating metal body depending on the desired duration of unilateral heating of details at given values for t_m, h и R/h;
– Creation of a scientifically derived model-based automatic control of the unilateral heating process.

The mathematical model could be input into the software of programmable controllers for model predictive automatic control (Deliiski, 2004; Hadjiski, 2013; Hadjiski and Deliiski, 2015) of unilateral heating of wood details used for different purposes after the bending of the details.

The methodology for the development and solution of the model could be further applied in the creation of analogous models, for example, for the calculation of the change of the temperature and energy consumption in flat details during their heating before lacquering (Deliiski *et al*., 2016a) or in equipment walls made of various materials.

Symbols – Simboli

a = temperature conductivity, $m^2 \cdot s^{-1}$
c = specific heat capacity, $J \cdot kg^{-1} \cdot K^{-1}$
h = thickness, m
M = number of steps on the x-coordinate used for solving the model
N = number of steps on the τ-coordinate used for solving the model
q = specific energy consumption, $kWh \cdot m^{-2}$
$dq/d\tau$ = specific heat flux, $kW \cdot m^{-2}$
R = radius, m
S = wood shrinkage, %
t = temperature, ºC: $t = T - 273.15$
T = temperature, K: $T = t + 273.15$
u = moisture content, $kg \cdot kg^{-1}$: $u = W/100$
W = moisture content, %: $W = 100u$
x = coordinate along the thickness of details: $0 \leq x \leq X = h$
α = heat transfer coefficient, $W \cdot m^{-2} \cdot K^{-1}$
λ = thermal conductivity, $W \cdot m^{-1} \cdot K^{-1}$
ρ = density, $kg \cdot m^{-3}$
τ = time, s
Δx = step on the x-coordinate of the model, which coincides with the thickness of wood detail subjected to heating, m
$\Delta\tau$ = step on the τ-coordinate for solution of the model, i.e. interval between time levels, s
Δq = change of q for time equal to $\Delta\tau$, $kWh \cdot m^{-2}$
@ = at

Subscripts / indeksi:

a = air (air temperature near the non-heated side of wood details)
avg = average (the average mass temperature of details at a given moment of their unilateral heating or the average arithmetic values of thermo physical characteristics of wood)
b = basic (density, based on dry mass divided by green volume)

e = emission (energy of heat flux needed for covering of the heat emission from the details)
fsp = fiber saturation point of wood
i = nodal point of the calculation mesh along detail thickness: i = 1, 2, 3, 4, 5, 6, 7, 8, 9
m = medium (temperature of the heating metal band used for unilateral heating)
p = process (duration of the whole process of unilateral heating)
total = total (energy or heat flux needed both for heating of wood and for covering of the heat emission from the details)
v = volume (wood shrinkage)
w = wood (energy or heat flux needed for heating of wood)
ws = wood surface (non-heated surface of wood details)
0 = initial (the average mass temperature of the details in the beginning of the heating or the time level in the beginning of the model's solution)

Superscripts / eksponenti:

n = time level: n = 0, 1, 2, 3, ..., N
293.15 = at 293.15 K, i.e. at 20 ºC (for the standard values of the wood fiber saturation point)

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